

Questions

20N.1.SL.TZ0.T_6

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

a.i. Find the probability that the first ball chosen is labelled A. [1]

a.ii. Find the probability that the first ball chosen is labelled A or labelled N. [1]

b.

Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N.

[2]

c. Find the probability that both balls chosen are labelled N. [2]

19M.2.SL.TZ1.S_10

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

Ted plays a game using these dice. The rules are:

Having a turn means to roll all three dice.

He wins \$10 for each green face rolled and adds this to his winnings.

After a turn Ted can either:

end the game (and keep his winnings), or

have another turn (and try to increase his winnings).

If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D , where $\$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$P(D = d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

a.i. Find the probability of rolling exactly one red face. [2]

a.ii. Find the probability of rolling two or more red faces. [3]

b.

Show that, after a turn, the probability that Ted adds exactly \$10 to his winnings is $\frac{1}{3}$.

[5]

c.i. Write down the value of x . [1]

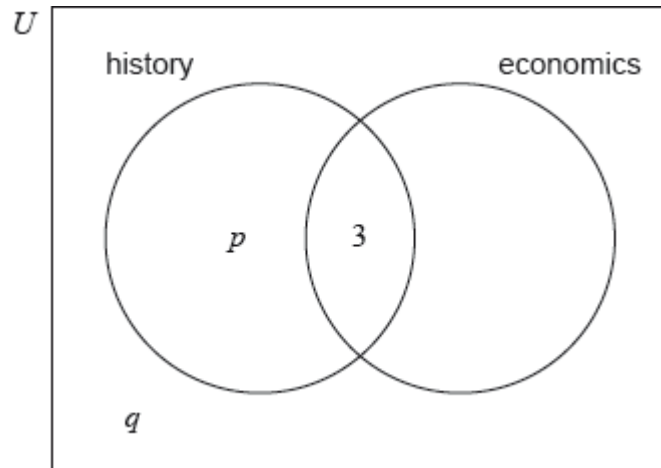
c.ii. Hence, find the value of y . [2]

d. Ted will always have another turn if he expects an increase to his winnings.

Find the least value of w for which Ted should end the game instead of having another turn.

[3]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



a.i. Find the value of p ; [2]

a.ii. Find the value of q . [2]

b.

A girl is selected at random. Find the probability that she takes economics but not history.

[2]

18N.1.SL.TZ0.S_9

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

Let $n = 5$. Find the probability that the game will end on her

a.i. Find the probability, in terms of n , that the game will end on her first draw. [1]

a.ii. Find the probability, in terms of n , that the game will end on her second draw.

[3]

b.i. third draw. [2]

b.ii. fourth draw. [2]

c.

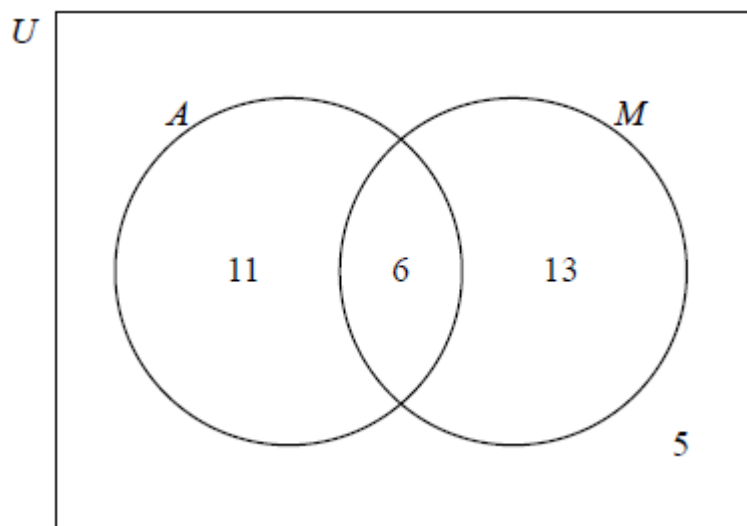
Hayley plays the game when $n = 5$. She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4
Money earned back (\$M)	0	20	$8k$	$12k$

Find the value of k so that this is a fair game. [7]

18N.2.SL.TZ0.S_1

In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.



One student from the group is chosen at random. Find the probability that

- a. Write down the number of students in the group who take art class. [2]
- b.i. the student does not take art class. [2]
- b.ii. the student takes either art class or music class, but not both. [2]

22M.1.SL.TZ2.9

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $E(X) = 2$.

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a , a and b , where $a < b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

a. Show that $p = 0.4$ and $q = 0.2$. [5]

b. Find $P(X > 2)$. [2]

c.

Assuming that rolls of the die are independent, find the probability that Nicky wins the game.

[5]

d. Determine the value of b . [2]

e. Find the value of a , providing evidence for your answer. [2]

SPM.2.SL.TZ0.8

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2 .

The probability that X is less than 24.15 is 0.1446.

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

a. Find $P(24.15 < X < 25)$. [2]

b.i. Find σ , the standard deviation of X . [3]

b.ii.

Hence, find the probability that a seashell selected at random has a length greater than 26 mm.

[2]

c. Find $E(Y)$. [3]

d.

Find the probability that exactly three of these seashells have a length greater than 26 mm.

[2]

e. A seashell selected at random has a length less than 26 mm.

Find the probability that its length is between 24.15 mm and 25 mm. [3]

21N.2.SL.TZ0.9

The random variable X follows a normal distribution with mean μ and standard deviation σ .

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ . Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

Find the probability that an avocado chosen at random from this purchase is categorized as

- a. Find $P\mu - 1.5\sigma < X < \mu + 1.5\sigma$. [3]
- b. Find the value of μ and of σ . [5]
- c.i. medium. [2]
- c.ii. large. [1]
- c.iii. premium. [1]
- d.

The selling prices of the different categories of avocado at this supermarket are shown in the following table:

Category	Medium	Large	Premium
Selling price (\$) per avocado	1.10	1.29	1.96

The supermarket pays the farm \$ 200 for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

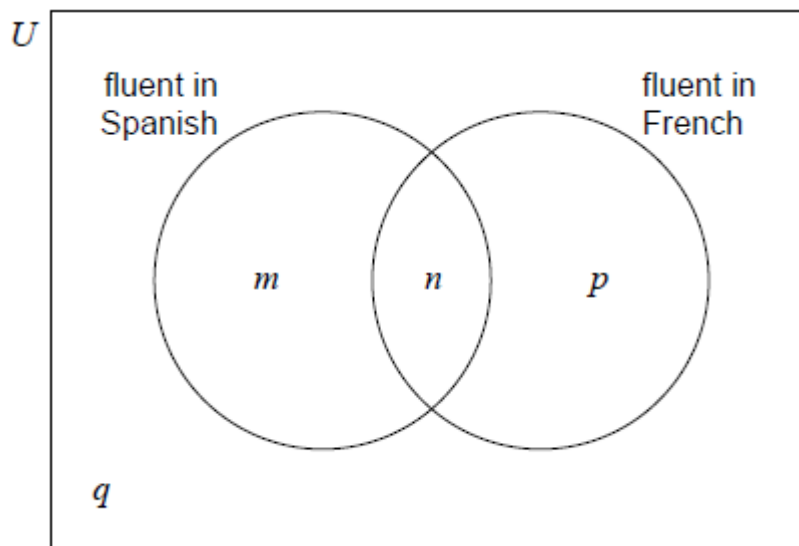
According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least \$ 438.

[4]

19N.1.SL.TZ0.S_2

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events "fluent in Spanish" and "fluent in French".

The values m , n , p and q represent numbers of students.



- Write down the value of q . [1]
- Find the value of n . [2]
- Write down the value of m and of p . [3]

21M.2.SL.TZ2.4

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

- a. Find the probability that the student plays a sport and is involved in theatre.

[2]

- b.

Find the probability that the student is involved in theatre, but does not play a sport.

[2]

- c. Find $P(G \cap T)$.

[2]

- d. Determine if the events G and T are independent. Justify your answer.

[2]

SPM.1.SL.TZ0.2

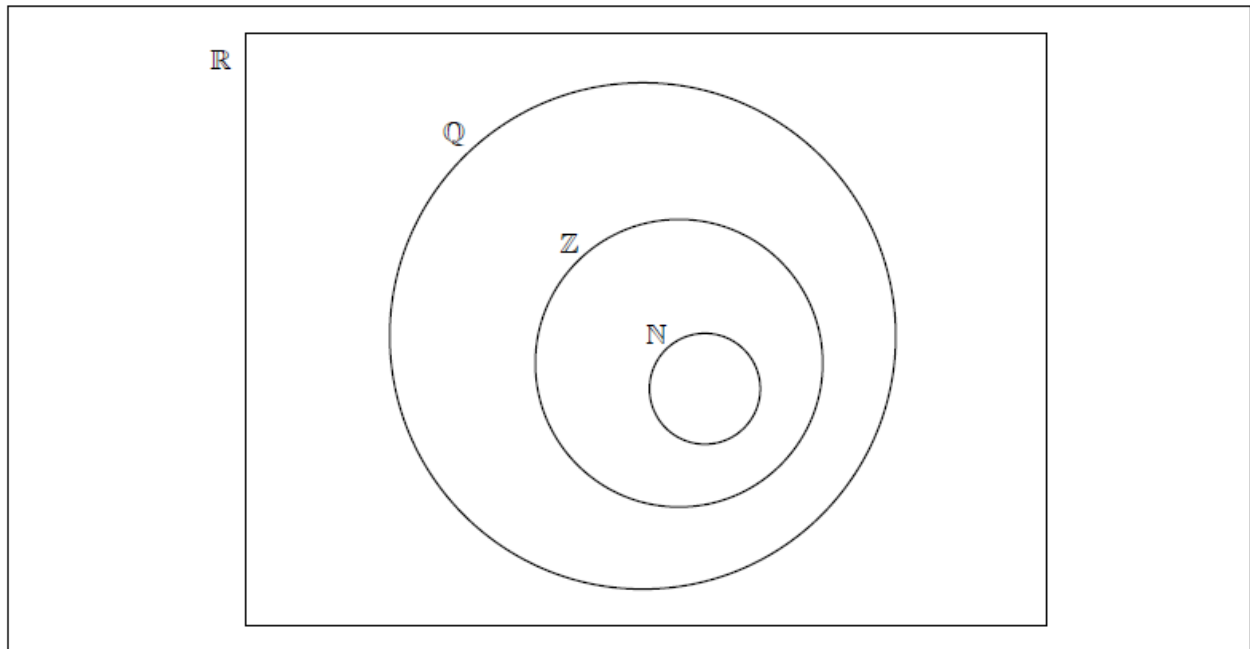
Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.

Find $P(A|B)$.

19M.1.SL.TZ1.T_7

a.

Place the numbers 2π , -5 , 3^{-1} and $2^{\frac{3}{2}}$ in the correct position on the Venn diagram.



[4]

b.

In the table indicate which **two** of the given statements are true by placing a tick (✓) in the right hand column.

Statement	True
$Z \subset Q$	
$N \subset Q'$	
$N \cap Z = N$	
$Q \cup R = Z'$	

[2]

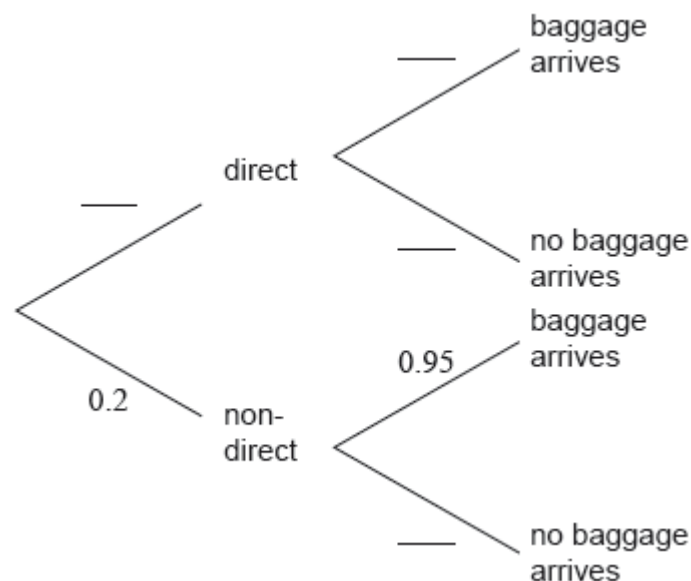
17M.1.SL.TZ1.T_7

Sara regularly flies from Geneva to London. She takes either a direct flight or a non-direct flight that goes via Amsterdam.

If she takes a direct flight, the probability that her baggage does not arrive in London is 0.01.

If she takes a non-direct flight the probability that her baggage arrives in London is 0.95.

The probability that she takes a non-direct flight is 0.2.

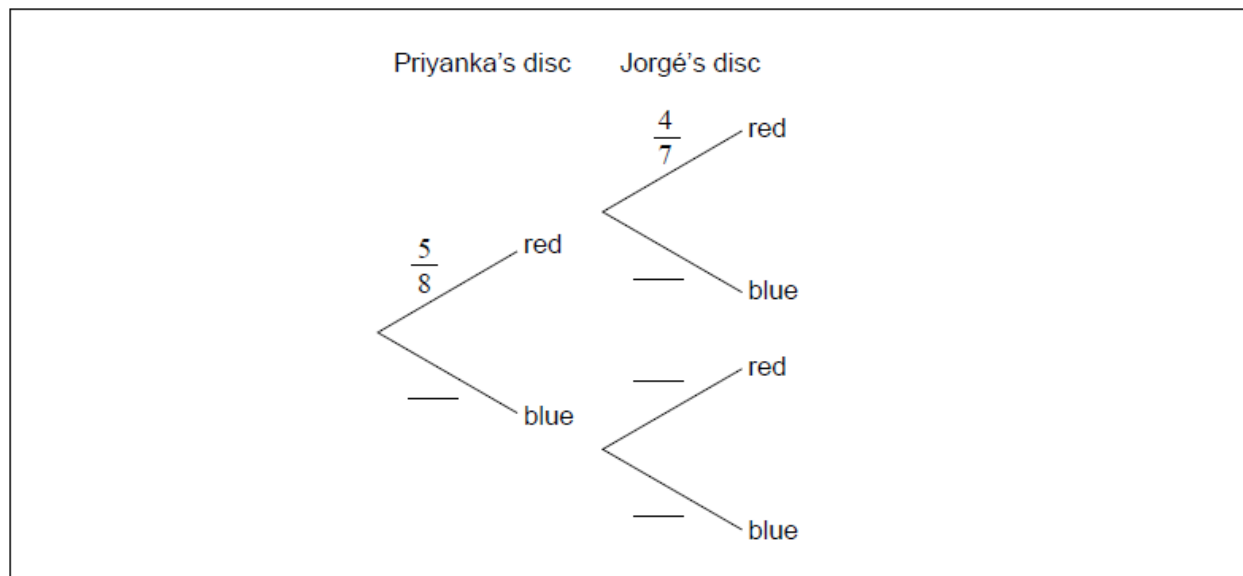


- Complete the tree diagram. [3]
- Find the probability that Sara's baggage arrives in London. [3]

18N.1.SL.TZ0.T_8

A bag contains 5 red and 3 blue discs, all identical except for the colour. First, Priyanka takes a disc at random from the bag and then Jorge takes a disc at random from the bag.

- a. Complete the tree diagram.



[3]

- b. Find the probability that Jorge chooses a red disc.

[3]

18M.1.SL.TZ2.T_9

Consider the following Venn diagrams.

Diagram 1

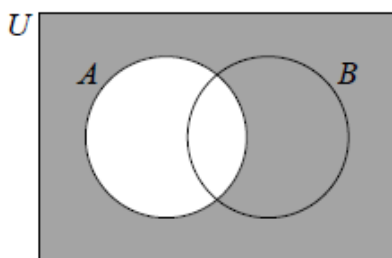


Diagram 2

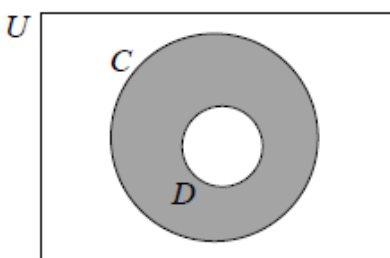
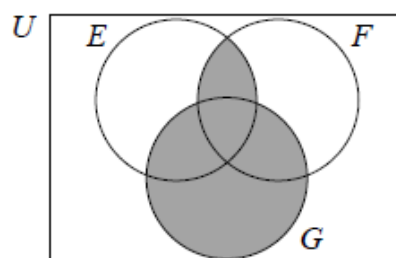


Diagram 3



a.i.

Write down an expression, in set notation, for the **shaded** region represented by Diagram 1.

[1]

a.ii.

Write down an expression, in set notation, for the **shaded** region represented by Diagram 2.

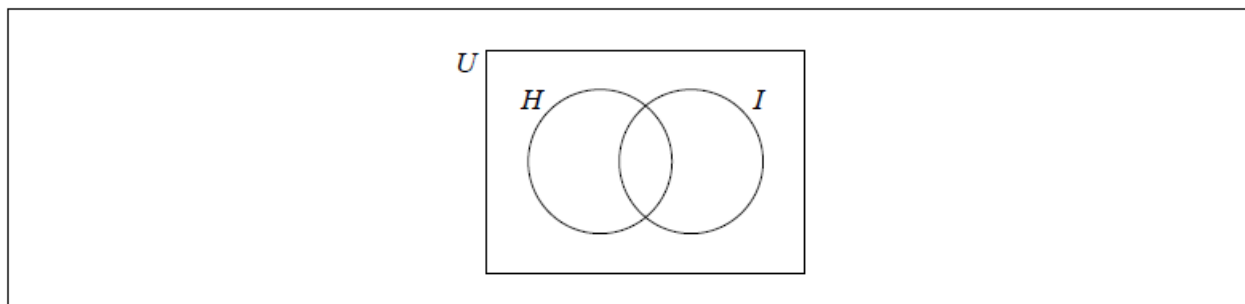
[1]

a.iii.

Write down an expression, in set notation, for the shaded region represented by Diagram 3.

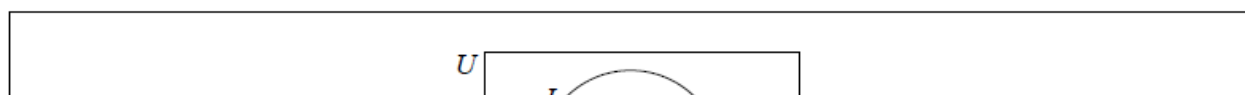
[2]

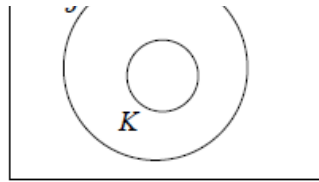
b.i. Shade, on the Venn diagram, the region represented by the set $(H \cup I)'$.



[1]

b.ii. Shade, on the Venn diagram, the region represented by the set $J \cap K$.





[1]

17N.2.AHL.TZ0.H_2

Events A and B are such that $P(A \cup B) = 0.95$, $P(A \cap B) = 0.6$ and $P(A | B) = 0.75$.

- a. Find $P(B)$. [2]
- b. Find $P(A)$. [2]
- c. Hence show that events A' and B are independent. [2]

20N.1.SL.TZ0.T_14

Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

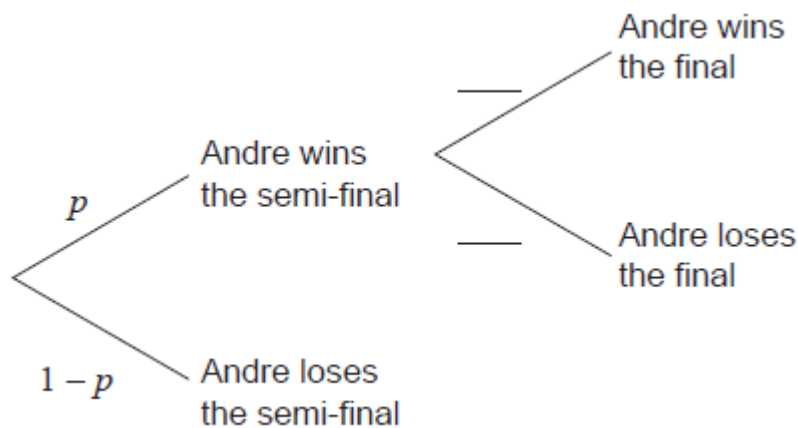
If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p . If Andre wins the semi-final, then the probability he will be the champion is 0.6.

The probability that Andre will not be the champion is 0.58.

- a. Complete the values in the tree diagram.

[1]



- b. Find the value of p .

[2]

c.

Given that Andre did not become the champion, find the probability that he lost in the semi-final.

[3]

19M.2.AHL.TZ1.H_9

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake.

The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

Find the probability that a customer chosen at random will buy

On a typical day 200 customers come to the café.

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

a.i. both a sandwich and a cake. [3]

a.ii. only a sandwich. [1]

b.i. Find the expected number of cakes sold on a typical day. [1]

b.ii. Find the probability that more than 100 cakes will be sold on a typical day.

[3]

c.i.

A customer is selected at random. Find the probability that the customer is male and buys a sandwich.

[1]

c.ii.

A female customer is selected at random. Find the probability that she buys a sandwich.

[4]

18M.2.AHL.TZ1.H_6

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

21N.1.SL.TZ0.4

Box 1 contains 5 red balls and 2 white balls.

Box 2 contains 4 red balls and 3 white balls.

a.

A box is chosen at random and a ball is drawn. Find the probability that the ball is red.

[3]

b.

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

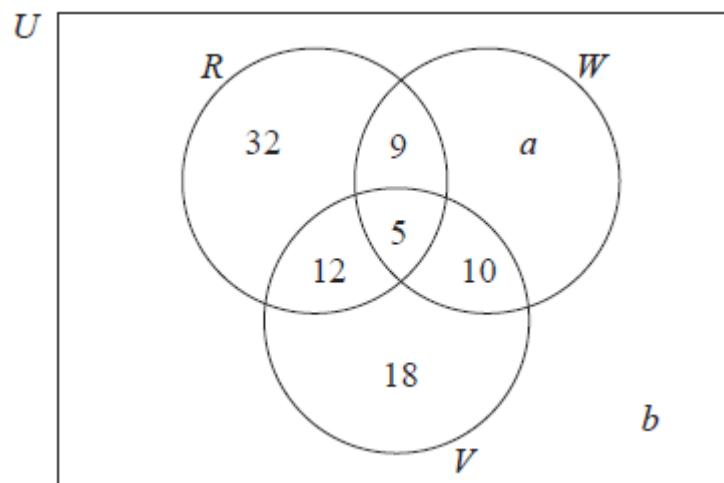
Determine whether events A and R are independent.

[2]

20N.2.SL.TZ0.T_2

On a school excursion, 100 students visited an amusement park. The amusement park's main attractions are rollercoasters (R), water slides (W), and virtual reality rides (V).

The students were asked which main attractions they visited. The results are shown in the Venn diagram.



A total of 74 students visited the rollercoasters or the water slides.

a.i. Find the value of a . [2]

a.ii. Find the value of b . [2]

b. Find the number of students who visited at least two types of main attraction. [2]

c. Write down the value of $n(R \cap W)$. [1]

d.i. Find the probability that a randomly selected student visited the rollercoasters. [2]

d.ii.

Find the probability that a randomly selected student visited the virtual reality rides. [1]

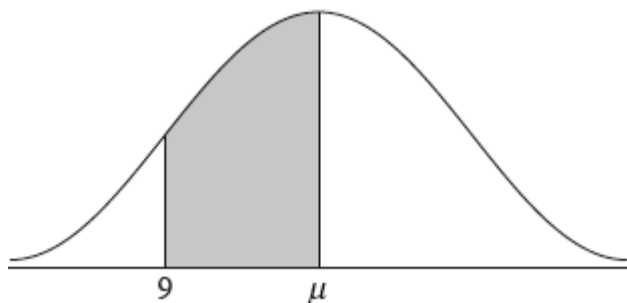
e.

Hence determine whether the events in **parts (d)(i)** and **(d)(ii)** are independent. Justify your reasoning.

[2]

17M.2.SL.TZ1.S_9

A random variable X is normally distributed with mean, μ . In the following diagram, the shaded region between 9 and μ represents 30% of the distribution.



The standard deviation of X is 2.1.

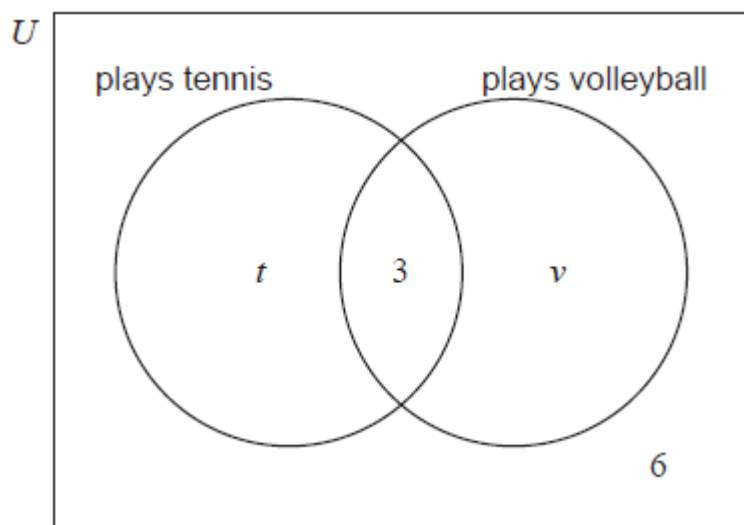
The random variable Y is normally distributed with mean λ and standard deviation 3.5. The events $X > 9$ and $Y > 9$ are independent, and $P((X > 9) \cap (Y > 9)) = 0.4$.

- Find $P(X < 9)$. [2]
- Find the value of μ . [3]
- Find λ . [5]
- Given that $Y > 9$, find $P(Y < 13)$. [5]

20N.1.SL.TZ0.S_1

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events "plays tennis" and "plays volleyball". The values t and v represent numbers of students.



a.i. Find the value of t . [2]

a.ii. Find the value of v . [2]

b.

Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both.

[2]

Markschemes

20N.1.SL.TZ0.T_6

a.i.

$\frac{3}{9}$ $\frac{1}{3}$, 0.333, 0.333333..., 33.3% (A1) (C1)

[1 mark]

a.ii. $\frac{5}{9}$ 0.556, 0.555555..., 55.6% (A1) (C1)

[1 mark]

b. $\frac{3}{8}$ 0.375, 37.5% (A1)(A1) (C2)

Note: Award (A1) for correct numerator, (A1) for correct denominator. [2 marks]

c. $\frac{2}{9} \times \frac{1}{8}$ (M1)

Note: Award (M1) for a correct compound probability calculation seen.

$\frac{2}{72}$ $\frac{1}{36}$, 0.0278, 0.0277777..., 2.78% (A1) (C2) [2 marks]

19M.2.SL.TZ1.S_10

a.i.

valid approach to find $P(\text{one red})$ (M1)

$$\text{eg } {}_n C_a \times p^a \times q^{n-a}, B(n, p), 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2, \left(\frac{3}{1}\right)$$

listing all possible cases for exactly one red (may be indicated on tree diagram)

$$P(1 \text{ red}) = 0.444 \left(= \frac{4}{9} \right) \quad [0.444, 0.445] \quad \mathbf{A1 \ N2}$$

[3 marks] [5 maximum for parts (a.i) and (a.ii)]a.ii. valid approach (M1) eg $P(X=2) + P(X=3)$, $1 - P(X \leq 1)$, $\text{binomcdf}\left(3, \frac{1}{3}, 2, 3\right)$

$$\text{correct working (A1) eg } \frac{2}{9} + \frac{1}{27}, 0.222 + 0.037, 1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9} \quad 0.259259$$

$$P(\text{at least two red}) = 0.259 \left(= \frac{7}{27} \right) \quad \mathbf{A1 \ N3}$$

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

b. recognition that winning \$10 means rolling exactly one green (M1)

recognition that winning \$10 also means rolling at most 1 red (M1)

eg "cannot have 2 or more reds" correct approach A1

$$\text{eg } P(1G \cap 0R) + P(1G \cap 1R), P(1G) - P(1G \cap 2R),$$

"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this A1 for purely numerical expressions.

one correct probability for their approach (A1)

$$\text{eg } 3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2, \frac{6}{27}, 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2, \frac{1}{9}, \frac{2}{9} \quad \text{correct working leading to } \frac{1}{3} \quad \mathbf{A1}$$

$$\text{eg } \frac{3}{27} + \frac{6}{27}, \frac{12}{27} - \frac{3}{27}, \frac{1}{9} + \frac{2}{9} \quad \text{probability} = \frac{1}{3} \quad \mathbf{AG \ N0 \ [5 marks]}$$

$$\text{c.i. } x = \frac{7}{27}, 0.259 \text{ (check FT from (a)(ii))} \quad \mathbf{A1 \ N1 \ [1 mark]}$$

c.ii. evidence of summing probabilities to 1 (M1)

$$\text{eg } \Sigma = 1, x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1, 1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$$

0.148147 (0.148407 if working with **their** x value to 3 sf)

$$y = \frac{4}{27} \text{ (exact), } 0.148 \quad \mathbf{A1 \ N2 \ [2 \ marks]}$$

d. correct substitution into the formula for expected value **(A1)**

$$\text{eg } -w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27} \quad \text{correct critical value (accept inequality)} \quad \mathbf{A1}$$

$$\text{eg } w = 34.2857 \left(= \frac{240}{7} \right), \quad w > 34.2857 \quad \$40 \quad \mathbf{A1 \ N2 \ [3 \ marks]}$$

17M.1.SL.TZ1.S_1

a.i.

valid approach **(M1)**

$$\text{eg } p + 3 = 13, 13 - 3$$

$$p = 10 \quad \mathbf{A1 \ N2}$$

[2 marks]

a.ii. valid approach **(M1)** eg $p + 3 + 5 + q = 20, 10 - 10 - 8 \quad q = 2 \quad \mathbf{A1 \ N2}$

[2 marks]

b. valid approach **(M1)** eg $20 - p - q - 3, 1 - \frac{15}{20}, n(E \cap H') = 5$

$$\frac{5}{20} \left(\frac{1}{4} \right) \quad \mathbf{A1 \ N2 \ [2 \ marks]}$$

18N.1.SL.TZ0.S_9

a.i.

$$\frac{2}{n} \quad \mathbf{A1 \ N1}$$

[1 mark]a.ii. correct probability for one of the draws **A1**

$$\text{eg } P(\text{not blue first}) = \frac{n-2}{n}, \text{ blue second} = \frac{2}{n-1} \quad \text{valid approach} \quad \mathbf{(M1)}$$

eg recognizing loss on first in order to win on second, $P(B' \text{ then } B)$, $P(B') \times P(B | B')$, tree diagram

$$\text{correct expression in terms of } n \quad \mathbf{A1 \ N3} \quad \text{eg } \frac{n-2}{n} \times \frac{2}{n-1}, \frac{2n-4}{n^2-n}, \frac{2(n-2)}{n(n-1)}$$

[3 marks]

$$\text{b.i. correct working} \quad \mathbf{(A1)} \quad \text{eg } \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{12}{60} \left(= \frac{1}{5} \right) \quad \mathbf{A1 \ N2} \quad \mathbf{[2 marks]}$$

$$\text{b.ii. correct working} \quad \mathbf{(A1)} \quad \text{eg } \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{6}{60} \left(= \frac{1}{10} \right) \quad \mathbf{A1 \ N2} \quad \mathbf{[2 marks]}$$

c. correct probabilities (seen anywhere) **(A1)(A1)**

$$\text{eg } P(1) = \frac{2}{5}, P(2) = \frac{6}{20} \quad (\text{may be seen on tree diagram})$$

valid approach to find $E(M)$ or expected winnings using **their** probabilities **(M1)**

$$\text{eg } P(1) \times (0) + P(2) \times (20) + P(3) \times (8k) + P(4) \times (12k),$$

$$P(1) \times (-20) + P(2) \times (0) + P(3) \times (8k - 20) + P(4) \times (12k - 20)$$

correct working to find $E(M)$ or expected winnings **(A1)**

$$\text{eg } \frac{2}{5}(0) + \frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k),$$

$$\frac{2}{5}(-20) + \frac{3}{10}(0) + \frac{1}{5}(8k - 20) + \frac{1}{10}(12k - 20)$$

correct equation for fair game **A1**

$$\text{eg } \frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k) = 20, \frac{2}{5}(-20) + \frac{1}{5}(8k - 20) + \frac{1}{10}(12k - 20) = 0$$

correct working to combine terms in k **(A1)**

$$\text{eg } -8 + \frac{14}{5}k - 4 - 2 = 0, 6 + \frac{14}{5}k = 20, \frac{14}{5}k = 14 \quad k = 5 \quad \mathbf{A1 \ NO}$$

Note: Do not award the final **A1** if the candidate's **FT** probabilities do not sum to 1.

[7 marks]

18N.2.SL.TZ0.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach **(M1)**

$$(A \cap M') + (A \cap M), \frac{17}{35}, 11 + 6$$

number of students taking art class = 17 **A1 N2**

[2 marks]

b.i. valid approach **(M1)** $13 + 5, 35 - 17, 18, 1 - P(A)$ 0.514285

$$P(A) = \frac{18}{35} \text{ (exact), } 0.514 \quad \mathbf{A1 N2} \quad \mathbf{[2 marks]}$$

b.ii. valid approach **(M1)** $11 + 13, 35 - 6 - 5, 24$ 0.685714

$$P(A \text{ or } M \text{ but not both}) = \frac{24}{35} \text{ (exact), } 0.686 \quad \mathbf{A1 N2} \quad \mathbf{[2 marks]}$$

22M.1.SL.TZ2.9

a.

uses $\sum PX = x = 1$ to form a linear equation in p and q (M1)correct equation in terms of p and q from summing to 1 A1

$$p + 0.3 + q + 0.1 = 1 \text{ OR } p + q = 0.6 \text{ (or equivalent)}$$

uses $EX = 2$ to form a linear equation in p and q (M1)correct equation in terms of p and q from $EX = 2$ A1

$$p + 0.6 + 3q + 0.4 = 2 \text{ OR } p + 3q = 1 \text{ (or equivalent)}$$

Note: The marks for using $\sum PX = x = 1$ and the marks for using $EX = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously A1

$$\text{for example, } 2q = 0.4 \Rightarrow q = 0.2 \text{ or } p + 3 \times 0.6 - p = 1 \Rightarrow p = 0.4$$

$$\text{so } p = 0.4 \text{ and } q = 0.2 \quad \mathbf{AG}$$

[5 marks]

b. valid approach (M1)

$$P(X > 2) = P(X = 3) + P(X = 4) \text{ OR } P(X > 2) = 1 - P(X = 1) - P(X = 2)$$

$$= 0.3 \quad \mathbf{A1} \quad \mathbf{[2 marks]}$$

c.

recognises at least one of the valid scores (6, 7, or 8) required to win the game (M1)

Note: Award **M0** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling 2, 4, 4, 2 or 3, 3

$$PT = 6 = 20 \cdot 30 \cdot 1 + 0.2^2 = 0.1 \quad \mathbf{A1}$$

$$\text{a score of 7 is obtained by rolling 3, 4 or 4, 3} \quad PT = 7 = 20 \cdot 20 \cdot 1 = 0.04 \quad \mathbf{A1}$$

a score of 8 is obtained by rolling 4, 4 $PT = 8 = 0.1^2 = 0.01$ **A1**

Note: The above 3 **A1** marks are independent of each other.

$P(\text{Nicky wins}) = 0.1 + 0.04 + 0.01 = 0.15$ **A1** **[5 marks]**

d. $3 + b = 8$ **(M1)** $b = 5$ **A1** **[2 marks]**

e. **METHOD 1 EITHER** $PS = 5 = \frac{4}{16}$ $PS = a + 2 = \frac{4}{16}$ **A1** $\Rightarrow a + 2 = 5$ **OR**

$PS = 6 = \frac{3}{16}$ $PS = a + 3 = \frac{2}{16}$ and $PS = 5 + 1 = \frac{1}{16}$ **A1** $\Rightarrow a + 3 = 6$ **OR**

$PS = 4 = \frac{3}{16}$ $PS = a + 1 = \frac{2}{16}$ and $PS = 1 + 3 = \frac{1}{16}$ **A1** $\Rightarrow a + 1 = 4$ **THEN**

$\Rightarrow a = 3$ **A1**

Note: Award **A0A0** for $a = 3$ obtained without working/reasoning/justification.

METHOD 2 EITHER correctly lists a relevant part of the sample space **A1**

for example, $S = 4 = 3, 1, 1, a, 1, a$ or $S = 5 = 2, a, 2, a, 2, a, 2, a$ or $S = 6 = 3, a, 3, a, 1, 5$

$a + 3 = 6$ eliminates possibilities (exhaustion) for $a < 5$ **OR**

convincingly shows that $a \neq 2, 4$ **A1**

$a \neq 4$, for example, $PS = 7 = \frac{2}{16}$ from 2, 5, 2, 5 and so $3, a, 3, a \Rightarrow a + 3 \neq 7$ **THEN**

$\Rightarrow a = 3$ **A1** **[2 marks]**

SPM.2.SL.TZ0.8

a.

attempt to use the symmetry of the normal curve **(M1)**eg diagram, $0.5 - 0.1446$

$$P(24.15 < X < 25) = 0.3554 \quad \mathbf{A1}$$

[2 marks]b.i. use of inverse normal to find z score **(M1)** $z = -1.0598$

$$\text{correct substitution } \frac{24.15 - 25}{\sigma} = -1.0598 \quad \mathbf{(A1)} \quad \sigma = 0.802 \quad \mathbf{A1} \quad \mathbf{[3 marks]}$$

$$\text{b.ii. } P(X > 26) = 0.106 \quad \mathbf{(M1)A1} \quad \mathbf{[2 marks]}$$

$$\text{c. recognizing binomial probability} \quad \mathbf{(M1)} \quad E(Y) = 10 \times 0.10621 \quad \mathbf{(A1)}$$

$$= 1.06 \quad \mathbf{A1} \quad \mathbf{[3 marks]}$$

$$\text{d. } P(Y = 3) \quad \mathbf{(M1)} = 0.0655 \quad \mathbf{A1} \quad \mathbf{[2 marks]}$$

$$\text{e. recognizing conditional probability} \quad \mathbf{(M1)} \quad \text{correct substitution} \quad \mathbf{A1}$$

$$\frac{0.3554}{1 - 0.10621} = 0.398 \quad \mathbf{A1} \quad \mathbf{[3 marks]}$$

21N.2.SL.TZ0.9

a.

$$P\frac{\mu - 1.5\sigma - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{\mu + 1.5\sigma - \mu}{\sigma} \quad (M1)$$

$$P-1.5 < Z < 1.5 \text{ OR } 1 - 2 \times PZ < -1.5 \quad (A1)$$

$$P-1.5 < Z < 1.5 = 0.866385...$$

$$P\mu - 1.5\sigma < X < \mu + 1.5\sigma = 0.866 \quad A1$$

Note: Do not award any marks for use of their answers from part (b).

[3 marks]

$$b. \quad z_1 = -1.75068... \text{ and } z_2 = 1.30468... \text{ (seen anywhere)} \quad (A1)$$

correct equations $(A1)(A1)$

$$\frac{106.2 - \mu}{\sigma} = -1.75068..., \quad \mu + 1.30468...\sigma = 182.6$$

attempt to solve their equations involving z values $(M1)$

$$\mu = 149.976..., \quad \sigma = 25.0051... \quad \mu = 150, \quad \sigma = 25.0 \quad A1 \quad [5 \text{ marks}]$$

c.i. new sample space is 96% (may be seen in (ii) or (iii)) $(M1)$

$$P_{\text{medium}} | \text{not small} \text{ OR } \frac{0.576}{0.96} \quad P_{\text{Medium}} = 0.6 \quad A1 \quad [2 \text{ marks}]$$

$$c.ii. \quad P_{\text{Large}} = 0.3 \quad A1 \quad [1 \text{ mark}]$$

$$c.iii. \quad P_{\text{Premium}} = 0.1 \quad A1 \quad [1 \text{ mark}]$$

d. attempt to express revenue from avocados $(M1)$

$$1.1 \times 0.6 + 1.29 \times 0.3 + 1.96 \times 0.1 \text{ OR } 1.243n$$

correct inequality or equation for net profit in terms of n $(A1)$

$$1.1 \times 0.6n + 1.29 \times 0.3n + 1.96 \times 0.1n - 200 \geq 438 \text{ OR } 1.243n - 200 = 438$$

attempt to solve the inequality $(M1)$ sketch OR $n = 513.274...$

$$n = 514 \quad A1$$

Note: Only award follow through in part (d) for 3 probabilities which add up to 1. FT of

probabilities from c) that do not add up to 1 should only be awarded **M** marks, where appropriate, in d).

[4 marks]

19N.1.SL.TZ0.S_2

a.

$$q = 5 \quad \mathbf{A1 \ N1}$$

[1 mark]

b. valid approach **(M1)** eg $(18 + 10 + 5) - 30$, $28 - 25$, $18 + 10 - n = 25$

$$n = 3 \quad \mathbf{A1 \ N2 \ [2 marks]}$$

c. valid approach for finding m or p (may be seen in part (b)) **(M1)**

$$\text{eg } 18 - 3, 3 + p = 10 \quad m = 15, p = 7 \quad \mathbf{A1A1 \ N3 \ [3 marks]}$$

21M.2.SL.TZ2.4

a.

EITHER

$$PS + PT + PS' \cap T' - PS \cap T = 1 \quad \text{OR} \quad PS \cup T = PS' \cap T' \quad (M1)$$

$$0.7 + 0.2 + 0.18 - PS \cap T = 1 \quad \text{OR} \quad PS \cup T = 1 - 0.18$$

ORa clearly labelled Venn diagram **(M1)****THEN**

$$PS \cap T = 0.08 \quad (\text{accept } 8\%) \quad \mathbf{A1}$$

Note: To obtain the **M1** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

b. **EITHER** $PT \cap S' = PT - PT \cap S = 0.2 - 0.08$ OR

$$PT \cap S' = PT \cup S - PS = 0.82 - 0.7 \quad (M1)$$

ORa clearly labelled Venn diagram including PS , PT and $PS \cap T$ **(M1)****THEN**

$$= 0.12 \quad (\text{accept } 12\%) \quad \mathbf{A1} \quad [2 \text{ marks}]$$

c. $PG \cap T = PT / GPG \quad 0.25 \times 0.48 \quad (M1) = 0.12 \quad \mathbf{A1} \quad [2 \text{ marks}]$

d. **METHOD 1** $PG \times PT = 0.48 \times 0.2 = 0.096 \quad \mathbf{A1}$

$$PG \times PT \neq PG \cap T \Rightarrow G \text{ and } T \text{ are not independent} \quad \mathbf{R1} \quad \mathbf{METHOD 2}$$

$$PT \mid G = 0.25 \quad \mathbf{A1} \quad PT \mid G \neq PT \Rightarrow G \text{ and } T \text{ are not independent} \quad \mathbf{R1}$$

Note: Do not award **A0R1**. **[2 marks]**

attempt to substitute into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(M1)**

Note: Accept use of Venn diagram or other valid method.

$$0.6 = 0.5 + 0.4 - P(A \cap B) \quad \textbf{(A1)}$$

$$P(A \cap B) = 0.3 \text{ (seen anywhere)} \quad \textbf{A1}$$

attempt to substitute into $P(A|B) = \frac{P(A \cap B)}{P(B)}$ **(M1)**

$$= \frac{0.3}{0.4}$$

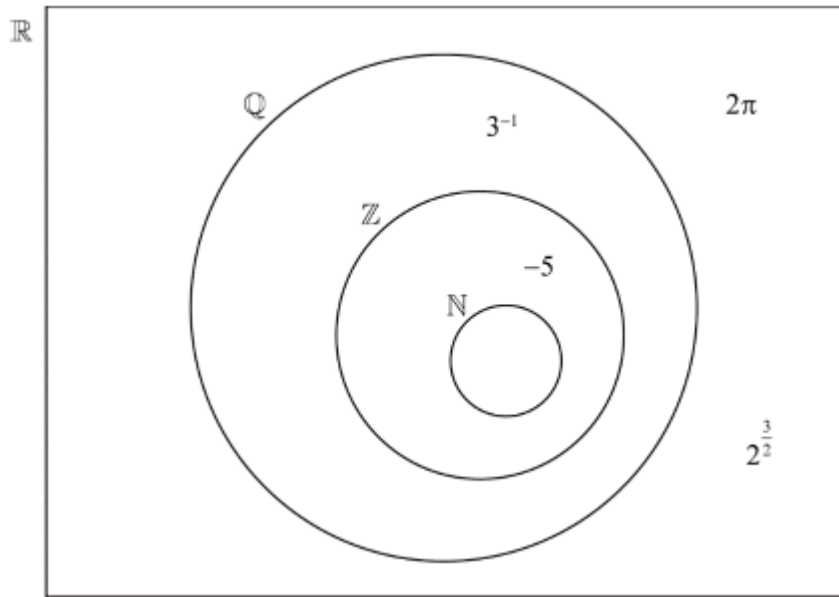
$$P(A|B) = 0.75 \left(= \frac{3}{4} \right) \quad \textbf{A1}$$

[5 marks]

19M.1.SL.TZ1.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1)(A1)(A1) (C4)

Note: Award (A1) for each number in the correct position.

[4 marks]

b.

Statement	True
$Z \subset Q$	✓
$N \subset Q'$	
$N \cap Z = N$	✓
$Q \cup R = Z'$	

Note: Award (A1) for each correctly placed tick.

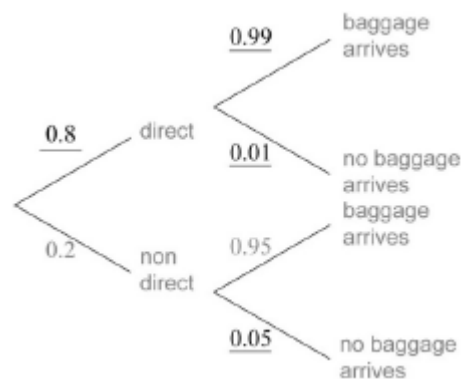
(A1)(A1) (C2)

[2 marks]

17M.1.SL.TZ1.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1)(A1) (C3)

Note: Award **(A1)** for each correct pair of probabilities.

[3 marks]

b. $0.8 \times 0.99 + 0.2 \times 0.95$ **(A1)(ft)(M1)**

Note: Award **(A1)(ft)** for two correct products of probabilities taken from their diagram, **(M1)** for the addition of their products.

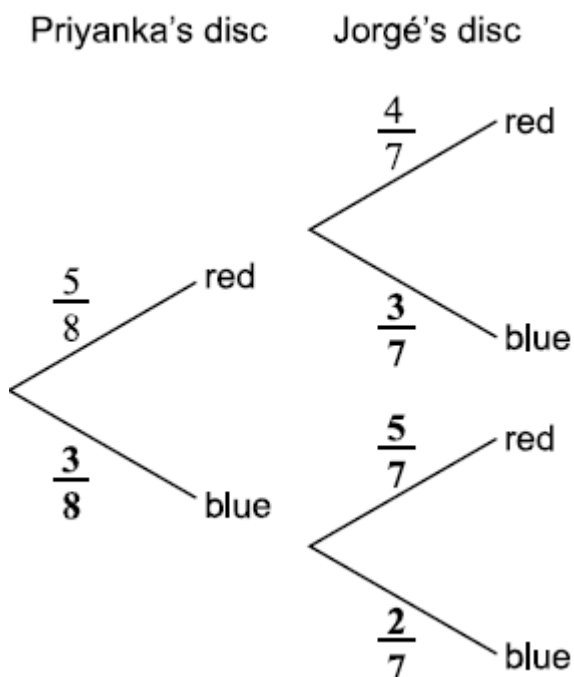
$= 0.982 \left(98.2\%, \frac{491}{500} \right)$ **(A1)(ft) (C3)** **Note:** Follow through from part (a).

[3 marks]

18N.1.SL.TZ0.T_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1)(A1) (C3)

Note: Award **(A1)** for each correct pair of branches.

[3 marks]

b. $\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7}$ **(A1)(ft)(M1)**

Note: Award **(A1)(ft)** for **their** two correct products from their tree diagram. Follow through from part (a), award **(M1)** for adding their two products. Award **(M0)** if additional products or terms are added.

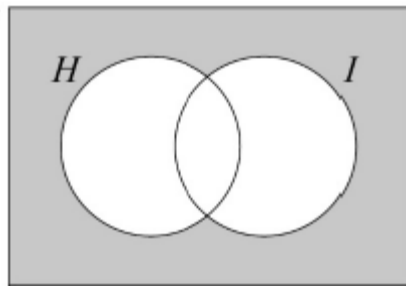
$= \frac{5}{8} \left(\frac{35}{56}, 0.625, 62.5\% \right)$ **(A1)(ft) (C3)**

Note: Follow through from their tree diagram, only if probabilities are [0,1].

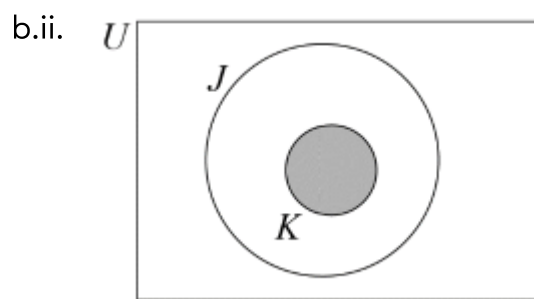
[3 marks]

18M.1.SL.TZ2.T_9

a.i.

 A' (A1)**Note:** Accept alternative set notation for complement such as $U - A$.**[1 mark]**a.ii. $C \cap D'$ OR $D' \cap C$ (A1) **Note:** Accept alternative set notation for complement.**[1 mark]**a.iii. $(E \cap F) \cup G$ OR $G \cup (E \cap F)$ (A2) (C4)**Note:** Accept equivalent answers, for example $(E \cup G) \cap (F \cup G)$. **[2 marks]**b.i. U **[1 mark]**

(A1)



(A1) (C2)

[1 mark]

17N.2.AHL.TZ0.H_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow 0.75 = \frac{0.6}{P(B)} \quad \textbf{(M1)}$$

$$\Rightarrow P(B) \left(= \frac{0.6}{0.75} \right) = 0.8 \quad \textbf{A1}$$

[2 marks]

$$\text{b. } P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow 0.95 = P(A) + 0.8 - 0.6 \quad \textbf{(M1)}$$

$$\Rightarrow P(A) = 0.75 \quad \textbf{A1 [2 marks]}$$

$$\text{c. } \textbf{METHOD 1} \quad P(A' | B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.2}{0.8} = 0.25 \quad \textbf{A1} \quad P(A' | B) = P(A') \quad \textbf{R1}$$

hence A' and B are independent **AG**

Note: If there is evidence that the student has calculated $P(A' \cap B) = 0.2$ by assuming independence in the first place, award **AOR0**.

METHOD 2 EITHER $P(A) = P(A | B)$ **A1 OR**

$$P(A) \times P(B) = 0.75 \times 0.80 = 0.6 = P(A \cap B) \quad \textbf{A1 THEN } A \text{ and } B \text{ are independent} \quad \textbf{R1}$$

hence A' and B are independent **AG METHOD 3**

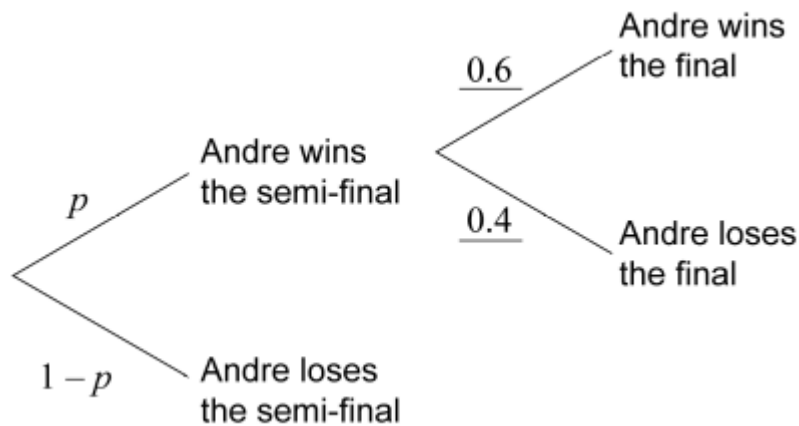
$$P(A') \times P(B) = 0.25 \times 0.80 = 0.2 \quad \textbf{A1} \quad P(A') \times P(B) = P(A' \cap B) \quad \textbf{R1}$$

hence A' and B are independent **AG [2 marks]**

20N.1.SL.TZ0.T_14

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

**(A1) (C1)**

Note: Award **(A1)** for the correct pair of probabilities.

[1 mark]

b. $p \times 0.4 + 1 - p = 0.58$ **(M1)**

Note: Award **(M1)** for multiplying and adding correct probabilities for losing equated to 0.58.

OR

$p \times 0.6 = 1 - 0.58$ **(M1)**

Note: Award **(M1)** for multiplying correct probabilities for winning equated to $1 - 0.58$ or 0.42.

$p = 0.7$ **(A1)(ft) (C2)**

Note: Follow through from their part (a). Award the final **(A1)(ft)** only if their p is within the range $0 < p < 1$.

[2 marks]

c. $\frac{0.3}{0.58} \frac{1-0.7}{0.58} \quad (A1)(ft) (A1)$

Note: Award **(A1)(ft)** for their correct numerator. Follow through from part (b). Award **(A1)** for the correct denominator.

OR

$\frac{0.3}{0.3 + 0.7 \times 0.4} \quad (A1)(ft) (A1)(ft)$

Note: Award **(A1)(ft)** for their correct numerator. Follow through from part (b). Award **(A1)(ft)** for their correct calculation of Andre losing the semi-final or winning the semi-final and then losing in the final. Follow through from their parts (a) and (b).

$\frac{15}{29} \quad 0.517, \quad 0.517241..., \quad 51.7\% \quad (A1)(ft) \quad (C3)$

Note: Follow through from parts (a) and (b). **[3 marks]**

19M.2.AHL.TZ1.H_9

a.i.

use of formula or Venn diagram (M1)

$$0.72 + 0.45 - 1 \quad (\mathbf{A1})$$

$$= 0.17 \quad \mathbf{A1}$$

[3 marks]

$$\text{a.ii. } 0.72 - 0.17 = 0.55 \quad \mathbf{A1} \quad [1 \text{ mark}]$$

$$\text{b.i. } 200 \times 0.45 = 90 \quad \mathbf{A1} \quad [1 \text{ mark}]$$

$$\text{b.ii. let } X \text{ be the number of customers who order cake } X \sim B(200, 0.45) \quad (\mathbf{M1})$$

$$P(X > 100) = P(X \geq 101) (= 1 - P(X \leq 100)) \quad (\mathbf{M1}) = 0.0681 \quad \mathbf{A1} \quad [3 \text{ marks}]$$

$$\text{c.i. } 0.46 \times 0.8 = 0.368 \quad \mathbf{A1} \quad [1 \text{ mark}]$$

$$\text{c.ii. METHOD 1 } 0.368 + 0.54 \times P(S|F) = 0.72 \quad \mathbf{M1A1A1}$$

Note: Award **M1** for an appropriate tree diagram. Award **M1** for LHS, **M1** for RHS.

$$P(S|F) = 0.652 \quad \mathbf{A1} \quad \text{METHOD 2 } P(S|F) = \frac{P(S \cap F)}{P(F)} \quad (\mathbf{M1})$$

$$= \frac{0.72 - 0.368}{0.54} \quad \mathbf{A1A1} \quad \text{Note: Award } \mathbf{A1} \text{ for numerator, } \mathbf{A1} \text{ for denominator.}$$

$$P(S|F) = 0.652 \quad \mathbf{A1} \quad [4 \text{ marks}]$$

18M.2.AHL.TZ1.H_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

X is number of squirrels in reserve

$X \sim \text{Po}(179.2)$ **A1**

Note: Award **A1** if 179.2 or 56×3.2 seen or implicit in future calculations.

recognising conditional probability **M1**

$P(X > 190 \mid X \geq 168)$

$$= \frac{P(X > 190)}{P(X \geq 168)} = \left(\frac{0.19827...}{0.80817...} \right) \quad \textbf{(A1)(A1)}$$

$= 0.245$ **A1**

[5 marks]

21N.1.SL.TZ0.4

a.

valid approach to find $P R$ **(M1)**

tree diagram (must include probability of picking box) with correct required probabilities

OR $PR \cap B_1 + PR \cap B_2$ OR $PR \mid B_1 P B_1 + PR \mid B_2 P B_2$

$$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2} \quad \textbf{(A1)}$$

$$P R = \frac{9}{14} \quad \textbf{A1}$$

[3 marks]

b. events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box **A2**

Note: Both conclusion and reasoning are required. Do not split the **A2**. **[2 marks]**

20N.2.SL.TZ0.T_2

a.i.

$$74 - 32 + 12 + 10 + 9 + 5 \quad \text{OR} \quad 74 - 68 \quad (\text{M1})$$

Note: Award **(M1)** for setting up a correct expression.

$$a = 6 \quad (\text{A1})(\text{G2})$$

[2 marks]

$$\text{a.ii. } 100 - 74 + 18 \quad (\text{M1}) \quad \text{OR} \quad 100 - 92 \quad (\text{M1}) \quad \text{OR}$$

$$100 - 32 + 9 + 5 + 12 + 10 + 18 + 6 \quad (\text{M1})$$

Note: Award **(M1)** for setting up a correct expression. Follow through from part (a)(i) but only for $a \geq 0$.

$$b = 8 \quad (\text{A1})(\text{ft})(\text{G2})$$

Note: Follow through from part(a)(i). The value of b must be greater or equal to zero for the **(A1)(ft)** to be awarded.

[2 marks]

$$\text{b. } 9 + 5 + 12 + 10 \quad (\text{M1})$$

Note: Award **(M1)** for adding 9, 5, 12 and 10.

$$36 \quad (\text{A1})(\text{G2}) \quad \text{[2 marks]}$$

$$\text{c. } 14 \quad (\text{A1}) \quad \text{[1 mark]}$$

$$\text{d.i. } \frac{58}{100} \quad \frac{29}{50}, \quad 0.58, \quad 58\% \quad (\text{A1})(\text{A1})(\text{G2})$$

Note: Award **(A1)** for correct numerator. Award **(A1)** for the correct denominator. Award **(A0)** for 58 only.

[2 marks]

$$\text{d.ii. } \frac{45}{100} \quad \frac{9}{20}, \quad 0.45, \quad 45\% \quad (\text{A1})(\text{ft})$$

100

20'

Note: Follow through from their denominator from part (d)(i). **[1 mark]**

e. they are not independent **(A1)(ft)** $\frac{58}{100} \times \frac{45}{100} \neq \frac{17}{100}$ **OR** $0.261 \neq 0.17$ **(R1)**

Note: Comparison of numerical values must be seen for **(R1)** to be awarded. **[2 marks]**
Do not award **(A1)(R0)**. Follow through from parts (d)(i) and (d)(ii).

17M.2.SL.TZ1.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach **(M1)**

eg $P(X < \mu) = 0.5, 0.5 - 0.3$

$P(X < 9) = 0.2$ (exact) **A1 N2**

[2 marks]

b. $z = -0.841621$ (may be seen in equation) **(A1)**

valid attempt to set up an equation with **their** z **(M1)**

eg $-0.842 = \frac{\mu - X}{\sigma}, -0.842 = \frac{X - \mu}{\sigma}, z = \frac{9 - \mu}{2.1}$ 10.7674 $\mu = 10.8$ **A1 N3 [3 marks]**

c. $P(X > 9) = 0.8$ (seen anywhere) **(A1)** valid approach **(M1)** eg $P(A) \times P(B)$

correct equation **(A1)** eg $0.8 \times P(Y > 9) = 0.4$ $P(Y > 9) = 0.5$ **A1**

$\lambda = 9$ **A1 N3 [5 marks]**

d. finding $P(9 < Y < 13) = 0.373450$ (seen anywhere) **(A2)**

recognizing conditional probability **(M1)** eg $P(A | B), P(Y < 13 | Y > 9)$

correct working **(A1)** eg $\frac{0.373}{0.5}$ 0.746901 0.747 **A1 N3 [5 marks]**

20N.1.SL.TZ0.S_1

a.i.

valid approach to find t **(M1)**

eg $t + 3 = 19$, $19 - 3$

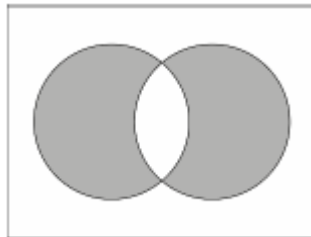
$t = 16$ (may be seen on Venn diagram) **A1 N2**

[2 marks]

a.ii. valid approach to find v **(M1)** eg $t + 3 + v + 6 = 30$, $30 - 19 - 6$

$v = 5$ (may be seen on Venn diagram) **A1 N2 [2 marks]**

b. valid approach **(M1)** eg $16 + 5$, 21 students, $1 - \frac{3+6}{30}$, $\frac{21}{30} = \frac{7}{10}$ **A1 N2**



[2 marks]